Why $2^{n-1}(2^n - 1)$?

Here is the story:

In the sixth century B.C., Pythagoras of Samos and his Pythagorean Brotherhood created a religion out of the study of numbers. They believed that nature operated according to mathematical secrets and that they could understand nature (and thus becomes godlike themselves) by unlocking those secret numerical relationships.

A major aspect of their fascination with numbers was the concept of a perfect number. A number is *perfect* if and only if it equals the sum of its proper divisors. d is a proper divisor of a natural number m if and only if $d|m$ and $1 \le d < m$. Thus 6 is perfect because $1 + 2 + 3 = 6$, whereas 10 is not perfect because $1 + 2 + 5 \ne 10$.

So what has all of this to do with $2^{n-1}(2^n - 1)$? Euclid (323–285 B.C.) proved the following theorem:

If n is a natural number such that $(2^n - 1)$ is prime, then $2^{n-1}(2^n - 1)$ is perfect.

Thus, $2^{n-1}(2^n - 1)$ is a tool in a quest for perfection.